HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate

Trial Examination Term 3 2022

STUDENT NUMBER: ____________________

STUDENT NAME: ____________________ **TEACHER NAME:** ____________________

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–12)

- Attempt Questions 11–14
- Start each question in a new writing booklet
- Write your student number on every writing booklet
- Allow about 1 hour and 45 minutes for this section

Question	*1-10*	*11*	*12*	*13*	*14*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

Question 1

What is the angle between the vectors $\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 5\underset{\sim}{i} - \underset{\sim}{j}$?

(A) $58 \cdot 909^\circ$

(B) $60 \cdot 255^\circ$

(C) $82 \cdot 582^\circ$

(D) $82 \cdot 875^\circ$

Question 2

The equation $x^3 + 2x^2 - 3x + 6 = 0$ has roots α, β and γ .
What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{3}$

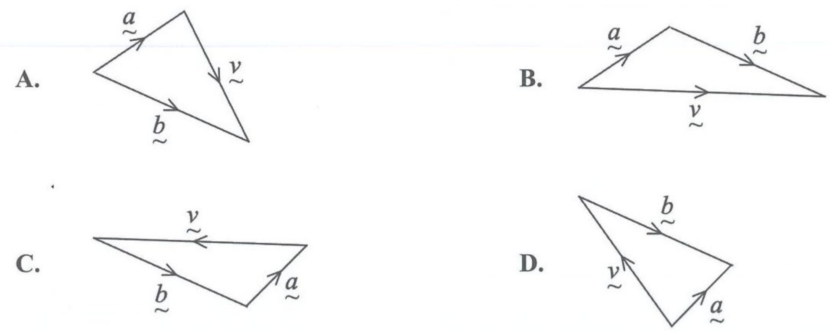
(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

Question 3

The vectors $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{b}$ are shown.

Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{b} - \underset{\sim}{a}$



Question 4

$\sqrt{3}\cos 2\theta - \sin 2\theta$ in the form $R\cos(2\theta+\alpha)$ where R is positive and α is in radians is

(A) $\frac{1}{2}\cos\left(2\theta+\frac{\pi}{6}\right)$

(B) $\frac{1}{2}\cos\left(2\theta+\frac{\pi}{3}\right)$

(C) $2\cos\left(2\theta+\frac{\pi}{6}\right)$

(D) $2\cos\left(2\theta+\frac{\pi}{3}\right)$

Question 5

$\sin(3x+x)-\sin(3x-x)=$

(A) $-2\sin 3x \sin x$

(B) $2\cos 3x \sin x$

(C) $2\cos 3x \cos x$

(D) $2\sin 3x \sin x$

Question 6

The vectors $\vec{p} = \begin{pmatrix} 4 \\ a+1 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} a \\ -2 \end{pmatrix}$ are perpendicular. What is the value of a ?

(A) -1

(B) 1

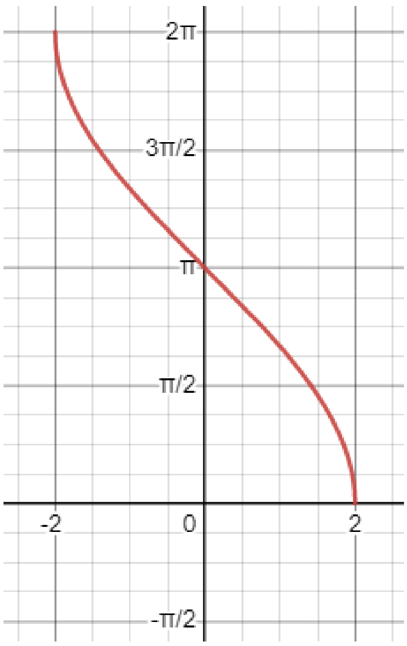
(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

Question 7

The function shown in the diagram below has equation $y = A\cos^{-1} Bx$.

Which of the following is true?



(A) $A = \frac{1}{2},\ B = \frac{1}{2}$

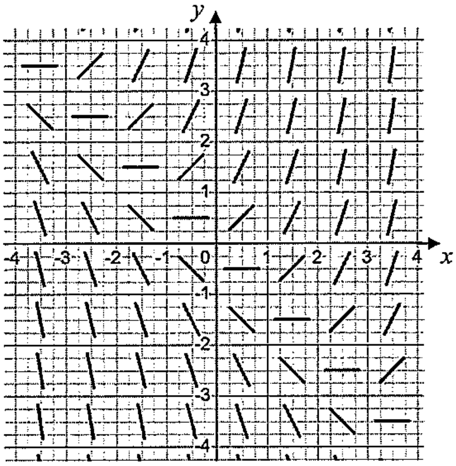
(B) $A = \frac{1}{2},\ B = 2$

(C) $A = 2,\ B = \frac{1}{2}$

(D) $A = 2,\ B = 2$

Question 8

Which differential equation is represented by the following slope field?



(A) $\frac{dy}{dx}+x+y=0$

(B) $\frac{dy}{dx}-x-y=0$

(C) $\frac{dy}{dx}+x-y=0$

(D) $\frac{dy}{dx}-x+y=0$

Question 9

What are the values of p for which $y=e^{1-px}$ satisfies the equation $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$?

(A) $p=1,\ p=2$

(B) $p=-1,\ p=2$

(C) $p=1,\ p=-2$

(D) $p=-1,\ p=-2$

Question 10

The members of a club votes for a new president. There were 12 candidates for the position of president and 1839 members voted. Each member voted for one candidate only.
One candidate received more votes than any other candidate and so became the new president.
What is the smallest number of votes the new president could have received?

(A) 156

(B) 155

(C) 154

(D) 153

Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section
Instructions

- Answer the questions in the appropriate writing booklet.
- In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) Consider the polynomial $P(x) = x(x-a) - b(b-a)$.

(i) Show that $(x-b)$ is a factor of $P(x)$. **1**

(ii) By division or otherwise, find the other factor. **2**

(b) Evaluate and find the exact value of $\int_{\sqrt{2}}^{\sqrt{6}} \frac{1}{2+x^2}\, dx$. **3**

(c) Find the area bounded by the $y = \cos^2(3x)$, the x-axis and the ordinates at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$. **3**

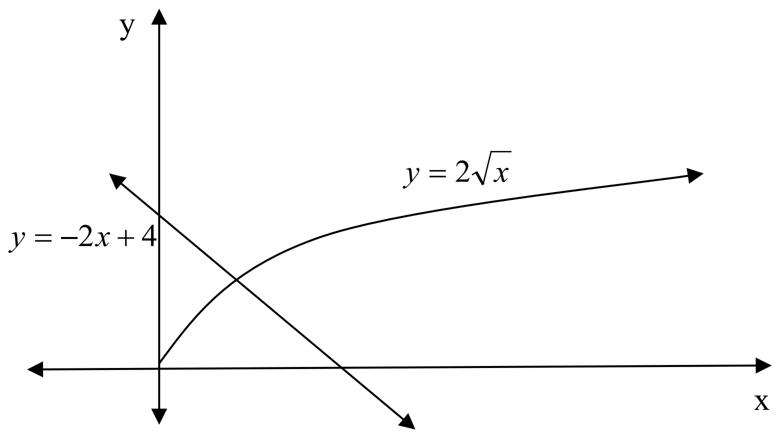
(d) Use the substitution $x = u^2 - 1 \quad (u > 0)$ to evaluate $\int_0^3 x\sqrt{x+1}\, dx$. **3**

(e) Prove, by Mathematical Induction, that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for all integers $n \geq 0$. **3**

End of Question 11

Question 12 (15 marks) Start a new writing booklet.

(a) The area enclosed by the curve $y = 2\sqrt{x}$, the line $y = -2x + 4$,
and the y axis is revolved about the y axis.



(i) Show $y = 2\sqrt{x}$ and $y = -2x + 4$ intersect at the coordinate $(1, 2)$. **1**

(ii) Find the volume of the solid of revolution about the y axis. **4**

(b) Consider the function $f(x) = \sec x$ over the domain $0 \le x < \dfrac{\pi}{2}$

(i) State the domain of $f^{-1}(x)$ **1**

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right)$ **1**

(iii) Hence find $\dfrac{d}{dx} f^{-1}(x)$ **2**

(c) A class consists of 10 girls and 8 boys. What is the probability of selecting a committee of 5, that contains 3 girls and 2 boys. **2**

(d) If $\overrightarrow{OA} = 2\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\overrightarrow{OB} = -4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{k} = \overrightarrow{OB} - \overrightarrow{OA}$ find the

(i) magnitude of $\underset{\sim}{k}$. **2**

(ii) direction of $\underset{\sim}{k}$ as a bearing, to the nearest degree. **2**

End of Question 12

Question 13 (15 marks) Start a new writing booklet.

(a) The rate of change of the share price $\$P$ of company $\boldsymbol{X}$ over a twelve month period can be modelled by the differential equation $\frac{dP}{dt} = k(P-6)$ where k is a constant and t is the time in months.

The price increases from \$10 per share to \$16 per share over 4 months.

(i) **By solving the differential equation** show that $P = 6 + 4e^{kt}$. **3**

(ii) After T months the share price is increasing at ten times the initial rate of increase. Find T correct to two decimal places. **3**

(b) A cubic block of ice of side length 8 cm is melting at a constant rate of $2\text{ cm}^3/\text{min}$. After time t minutes the cubic block of ice has edge length x cm and volume V cm^3.

(i) Show that $\frac{dx}{dt} = \frac{-2}{3x^2}$ **1**

(ii) Hence find an expression for x as a function of t. **3**

(c) (i) Show that $\frac{d}{dx}\left(\frac{2x}{4+x^2} + \tan^{-1}\frac{x}{2}\right) = \frac{16}{\left(4+x^2\right)^2}$ **3**

(ii) Hence, evaluate $\int_0^{2\sqrt{3}} \frac{dx}{\left(4+x^2\right)^2}$ **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet.

(a) If $f'(x) = \cot x + x$ and $f\left(\frac{\pi}{2}\right) = 0$, find an expression for $f(x)$. **3**

(b)

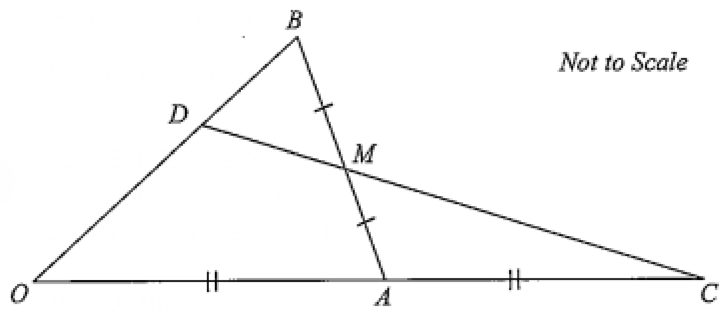
(i) Show that $\cos 4x = 8\left(\cos^4 x - \cos^2 x\right) + 1$. **2**

(ii) Hence, or otherwise, solve $\cos^2 x - \cos^4 x = \frac{1}{16}$ for $0 \le x \le \frac{\pi}{2}$. **2**

(c) For what values of x $(x \ne 0)$ does the geometric series

$1 + \frac{2x}{x+1} + \left(\frac{2x}{x+1}\right)^2 + \ldots$ have a limiting sum? **3**

(d)



In the diagram, side OA of ΔOAB is produced to C so that $AC = OA$.

M is the midpoint of AB and CM produced meets OB in D.

Given $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{CD} = \lambda \overrightarrow{CM}$ for some constant λ.

(i) Show that $\overrightarrow{OD} = \left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda \underset{\sim}{b}$. **3**

(ii) Hence show that $OD : DB = 2 : 1$ using $\overrightarrow{OD} = \mu \overrightarrow{OB}$ for some constant μ. **2**

End of paper

Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial 2022
Solutions

Multiple Choice

Solutions	Marker's Comments
Question 1 D $\cos\theta = \dfrac{1\times 5+3(-1)}{\sqrt{10}\times\sqrt{26}}$ $= \dfrac{2}{\sqrt{260}}$ $\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{260}}\right)$ $\approx 82.875°$	
Question 2 D $\alpha+\beta+\gamma = \dfrac{-b}{a} = -2$ $\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{c}{a} = -3$ $\alpha\beta\gamma = \dfrac{-d}{a} = -6$ $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma} = \dfrac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ $= \dfrac{-3}{-6}$ $= \dfrac{1}{2}$	
Question 3 A A. $\underset{\sim}{a}+\underset{\sim}{v}=\underset{\sim}{b}$ $\underset{\sim}{v}=\underset{\sim}{b}-\underset{\sim}{a}$ B. $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{v}$ C. $\underset{\sim}{b}+\underset{\sim}{a}=-\underset{\sim}{v}$ $\underset{\sim}{v}=-\underset{\sim}{a}-\underset{\sim}{b}$ D. $\underset{\sim}{v}+\underset{\sim}{b}=\underset{\sim}{a}$ $\underset{\sim}{v}=\underset{\sim}{a}-\underset{\sim}{b}$	

Solutions	Marker's Comments
Question 4 C $\sqrt{3}\cos 2\theta - \sin\theta = R\cos(2\theta+\alpha)$ $= R\cos 2\theta\cos\alpha - R\sin 2\theta\sin\alpha$ $R\cos\alpha = \sqrt{3},\ \ R\sin\alpha = 1$ $\tan\alpha = \frac{1}{\sqrt{3}},\ \ \alpha = \frac{\pi}{6}$ $R^2\sin^2\alpha + R^2\cos^2\alpha = 1 + \sqrt{3}^2$ $\mathrm{R}^2 = 4$ $R = 2$	
Question 5 B $\sin(A+B) - \sin(A-B) = 2\sin A\cos B$ $\sin(3x+x) - \sin(3x-x) = 2\sin 3x\cos x$	
Question 6 B $\begin{pmatrix} 4 \\ a+1 \end{pmatrix} \cdot \begin{pmatrix} a \\ -2 \end{pmatrix} = 0$ $4a - 2(a+1) = 0$ $2a - 2 = 0$ $a = 1$	
Question 7 C $-2 \le x \le 2$ $-1 \le \frac{x}{2} \le 1$ $0 \le y \le 2\pi$ $0 \le \frac{y}{2} \le \pi$ $\frac{y}{2} = \cos^{-1}\left(\frac{x}{2}\right)$ $y = 2\cos^{-1}\left(\frac{x}{2}\right)$ $A = 2, B = \frac{1}{2}$	

Solutions	Marker's Comments
Question 8 B $\frac{dy}{dx}=0$ for $y=-x$, exclude C and D $x>0, y>0, \frac{dy}{dx}>0$, exclude A or: choose two points to test, eg at $\left(\frac{1}{2},\frac{1}{2}\right)$, $\frac{dy}{dx}\approx 1$ and at $\left(-\frac{1}{2},\frac{1}{2}\right)$, $\frac{dy}{dx}$=0 only B is true for both points.	

Solutions	Marker's Comments
Question 9 C $y=e^{1-px}$ $\frac{dy}{dx}=-pe^{1-px}$ $\frac{d^2y}{dx^2}=p^2e^{1-px}$ $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$ $p^2e^{1-px}--pe^{1-px}-2\left(e^{1-px}\right)=0$ $\left(p^2+p-2\right)e^{1-px}=0$ $p^2+p-2=0 \quad \left(e^{1-px}\neq 0\right)$ $(p+2)(p-1)=0$ $p=-2,1$	
Question 10 B $1839\div 12=153.25$ All 12 candidates could receive 153 votes each. $153\times 12=1836$ with 3 more members to vote. If they vote for 3 different candidates, there is no clear winner. $\therefore$ 154 is not enough, 155 is needed.	Answer C was a common error

SECTION II

Solutions	Marker's Comments
Question 11 (a) (i) $P(b) = b(b-a) - b(b-a)$ $\quad = 0$ $\therefore (x-b)$ is a factor of $P(x)$.	You can use the factor's theorem or factorising in pair to show the factor. Generally well done.
(a) (ii) $$\begin{array}{r} \boldsymbol{x+(b-a)} \\ (\boldsymbol{x-b})\overline{\big)\ \boldsymbol{x^2 - ax - b(b-a)}} \\ \underline{\boldsymbol{x^2 - bx}} \qquad\qquad \\ \boldsymbol{(b-a)x - b(b-a)} \\ \underline{\boldsymbol{(b-a)x - b(b-a)}} \\ 0 \end{array}$$ $\therefore$ The other factor is $(\boldsymbol{x+b-a})$.	You can do long division or use factorising in part a). Generally well done.
(b) $\int_{\sqrt{2}}^{\sqrt{6}} \frac{1}{2+x^2}\,dx = \frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{x}{\sqrt{2}}\right]_{\sqrt{2}}^{\sqrt{6}}$ $= \frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{\sqrt{6}}{\sqrt{2}} - \tan^{-1}\frac{\sqrt{2}}{\sqrt{2}}\right)$ $= \frac{1}{\sqrt{2}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}1\right)$ $= \frac{1}{\sqrt{2}}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ $= \frac{1}{\sqrt{2}} \times \frac{\pi}{12}$ $= \frac{\sqrt{2}}{24}\pi$	Some students didn't find the inverse trig as the integral.

Question 11 (c) $2\cos^2(3x)-1=\cos(6x)$ $2\cos^2(3x)=\frac{1}{2}(\cos(6x)+1)$ $A=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos^2(3x)\,dx$ $=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{2}(\cos(6x)+1)\,dx$ $=\frac{1}{2}\left[\frac{\sin(6x)}{6}+x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\frac{1}{12}\left[\sin(6x)+6x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\frac{1}{12}\left(\sin\left(6\times\frac{\pi}{3}\right)+6\times\frac{\pi}{3}-\sin\left(6\times\frac{\pi}{6}\right)-6\times\frac{\pi}{6}\right)$ $=\frac{1}{12}(\sin 2\pi-\sin\pi+2\pi-\pi)$ $=\frac{1}{12}(0-0+\pi)$ $=\frac{\pi}{12}$	Some students didn't use the double angle formula, hence found the wrong integral.
(d) $x=u^2-1$ $\frac{dx}{du}=2u\ \therefore\ dx=2udu$ when $x=0, u=1 (u>0)$ when $x=3, u=2 (u>0)$ $\int_0^3 x\sqrt{x+1}\ dx=\int_1^2\left(u^2-1\right)\sqrt{u^2}\,2udu$ $=2\int_1^2\left(u^2-1\right)u^2du \quad (u>0)$ $=2\left[\frac{u^5}{5}-\frac{u^3}{3}\right]_1^2$ $=2\left(\frac{2^5}{5}-\frac{2^3}{3}-\frac{1^5}{5}+\frac{1^3}{3}\right)$ $=\frac{116}{15}$	**Some mistakes doing the substitution. To integrate in terms of u, the bourndaries need to be converted into the u values.**

Question 11 (e) For $n=0$, $5^{2n+1}+2^{2n+1}=5+2$ $=7$ which is divisible by 7. $\therefore$ Proven true for $n=0$. Assume true for $n=k$, i.e. $5^{2k+1}+2^{2k+1}$ is divisible by 7. i.e. $5^{2k+1}+2^{2k+1}=7P$ where $P\in\mathbb{R}$ Required to prove true for $n=k+1$, i.e. $5^{2(k+1)+1}+2^{2(k+1)+1}$ is divisible by 7. Proof: $5^{2(k+1)+1}+2^{2(k+1)+1}=5^{(2k+1)+2}+2^{(2k+1)+2}$ $=5^2\times5^{2k+1}+2^2\times2^{2k+1}$ $=25(7P-2^{2k+1})+4\times2^{2k+1}$ by assumption $=25\times7P-25\times2^{2k+1}+4\times2^{2k+1}$ $=25\times7P-21\times2^{2k+1}$ $=7(25P-3\times2^{2k+1})$ which is divisible by 7. $\therefore$ Proven true for $n=k+1$. If true for $n=k$, proven true for $n=k+1$. Since true for $n=0$, true for $n=0+1$, $n=1+1$,.... therefore, true for all integer $n(n\geq0)$.	Many missed the initial case of n=0. Some didn't show enough proof for n=k+1 by substitution.

Solutions	**Marker's Comments**
Question 12 (a) (i) $2\sqrt{x} = -2x + 4$ $4x = (-2x+4)^2$ $4x = 4x^2 - 16x + 16$ $4x^2 - 20x + 16 = 0$ $x^2 - 5x + 4 = 0$ $(x-4)(x-1) = 0$ $x = 1, 4$ sub into $y = -2x + 4$ $x = 1, y = -2(1) + 4 = 2$ $x = 4, y = -2(4) + 4 = -4$ since $y = 2\sqrt{x} > 0$, $\therefore (1,2)$ is the only point of intersection. **Alternatively, just show (1,2) satisfies both equations**	**1 mark** for a correct method with correct working. Most took the long path to answer this question rather than just sub (1,2) to show it's a solution.
(a) (ii) $y = 2\sqrt{x},\ x = \left(\frac{y}{2}\right)^2$ $y = -2x + 4,\ x = \frac{4-y}{2}$ $V = \pi\int_0^2 x^2 dy + \pi\int_2^4 x^2 dy$ $= \pi\int_0^2 \left(\frac{y}{2}\right)^4 dy + \pi\int_2^4 \left(\frac{4-y}{2}\right)^2 dy$ $= \frac{\pi}{16}\left[\frac{y^5}{5}\right]_0^2 + \frac{\pi}{4}\left[\frac{(4-y)^3}{-3}\right]_2^4$ $= \frac{\pi}{80}(32-0) - \frac{\pi}{12}(0-8)$ $= \frac{16\pi}{15}$	4 marks. This question was a rotation about the Y AXIS. Many did this about the x axis. When students did do it around the y axis the most common mistake was not using the correct boundaries for each part of the integral. Some students subtracted the integrals instead of adding them as well. Marks were awarded for change of subject to x bounds correct set up correct integrations correct substitution/solution
(b) (i) for $f(x) = \sec x$: Range: $y \geq 1$ for $y = f^{-1}(x)$: Domain: $x \geq 1$	Many failed to recognise there was a domain for the question and did not know what sec x looked like in this domain.

Solutions	**Marker's Comments**
Question 12 (b) (ii) $y = \sec x$ for $f^{-1}x$: $x = \sec y$ $= \dfrac{1}{\cos y}$ $\cos y = \dfrac{1}{x}$ $\therefore y = \cos^{-1}\left(\dfrac{1}{x}\right)$	1 mark Cosy=1/x was the crucial step in this question. Could have been done a lot better
(b) (iii) $\dfrac{dy}{dx} = \dfrac{-(-x^{-2})}{\sqrt{1-\left(\dfrac{1}{x}\right)^2}}$ $= \dfrac{x^{-2}}{\sqrt{\dfrac{x^2-1}{x^2}}}$ $= \dfrac{1}{x^2 \dfrac{1}{\lvert x \rvert}\sqrt{x^2-1}}$ $\quad but\ \lvert x \rvert = x\ for\ x > 1$ $= \dfrac{1}{x\sqrt{x^2-1}}$	Generally well done question except for proper simplification by quite a few students. Some integrated the 1/x to get ln(x) rather than differentiating to get -1/x^2 in the chain rule. The use of the absolute of x was ignored in the simplification Marks Correct use of chain rule with inverse trig Correct simplification
(c) $\dfrac{{}^{10}C_3 \times {}^{8}C_2}{{}^{18}C_5} = \dfrac{120 \times 28}{8568}$ $= \dfrac{20}{51}$	Generally well done question. A few silly mistakes caused loss of marks here. Some used addition in the numerator rather than multiplication Marks Correct numerator Correct denominator
(d) (i) $\overrightarrow{OB} - \overrightarrow{OA} = -4\underset{\sim}{i} - \underset{\sim}{j} - (2\underset{\sim}{i} - 4\underset{\sim}{j})$ $= -6\underset{\sim}{i} + 3\underset{\sim}{j}$ $\sqrt{36+9} = \sqrt{45}$ $= 3\sqrt{5}$	Generally, well done. No issues here. Marks Correct k vector Correct magnitude of k vector

Solutions	Marker's Comments
Question 12 (d) (ii) $\tan\theta = \frac{3}{6}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ direction in bearing: $270 + \tan^{-1}\left(\frac{1}{2}\right) = 296.5650...^\circ$ $\approx 297^\circ \boldsymbol{T}$	This on the whole was well done, however there were quite a few who attempted to find an angle between 2 vectors for some reason. This does not give a bearing that was required. Some found the correct angle of 27 degrees but misused it in finding the bearing. DRAW a diagram to help. Marks Correct angle Correct bearing
Question 13 (a) (i) $\frac{dt}{dP} = \frac{1}{k(P-6)}$ $dt = \frac{dP}{k(P-6)}$ $\int dt = \int \frac{dP}{k(P-6)}$ $t = \frac{1}{k}\ln\lvert P-6\rvert + C$ $t = 0, P = 10$ $0 = \frac{1}{k}\ln\lvert 10-6\rvert + C$ $C = -\frac{1}{k}\ln 4$ $t = \frac{1}{k}\ln\lvert P-6\rvert - \frac{1}{k}\ln 4$ $kt = \ln\left(\frac{\lvert P-6\rvert}{4}\right)$ $\frac{P-6}{4} = e^{kt}$ $P = 4e^{kt} + 6$ OR	1 for integrating 1 for finding c 1 for correct rearrangement

$\frac{dt}{dP}=\frac{1}{k(P-6)}$ $kdt=\frac{dP}{(P-6)}$ $\int kdt=\int\frac{1}{P-6}dP$ $kt=\ln\|P-6\|+C$ $when, t=0, P=10$ $0=\ln\|10-6\|+C$ $C=-\ln 4$ $kt=\ln\|P-6\|-\frac{1}{k}\ln 4$ $kt=\ln\left(\frac{\|P-6\|}{4}\right)$ $\frac{P-6}{4}=e^{kt}$ $P=4e^{kt}+6$ OR $\ln\|P-6\|=kt$ $P-6=e^{kt}$ $P-6=e^{kt}\times e^{c}$ $P-6=Ae^{kt} where A=\pm e^{c}$ $p=6+Ae^{kt}$ $When, t=0, P=10$ $10=6+Ae^{0}$ $10=6+A$ $A=4$ $P=6+4e^{kt}$	Some students did not solve the differential equation $\frac{dP}{dt}=k(P-6)$ to show that $P=4e^{kt}+6$, rather, they verified that $P=4e^{kt}+6$ is a solution of the DE, which is easier but NOT what the question asked for, so only one mark was awarded.

Solutions	Marker's Comments
Question 13 (a) (ii) $t=4, P=16$ $16=4e^{4k}+6$ $10=4e^{4k}$ $e^{4k}=\frac{10}{4}=\frac{5}{2}$ $4k=\ln\left(\frac{5}{2}\right)$ $k=\frac{1}{4}\ln\left(\frac{5}{2}\right)$	1 for finding value of k
$P=4e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)t}+6=4e^{kt}+6$ $\frac{dP}{dt}=4\left(\frac{1}{4}\ln\left(\frac{5}{2}\right)\right)e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)t}=4ke^{kt}$ $=\ln\left(\frac{5}{2}\right)e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)t}$ The initial rate of change is: $t=0, \frac{dP}{dt}=\ln\left(\frac{5}{2}\right)=4k\cong 0.916$ $when, t=T, \frac{dP}{dt}=10\times\ln\left(\frac{5}{2}\right)=40k$	Some students rounded off too early. Keep exact until final calculation.
$\therefore\ findTwhen$ $40k=4ke^{kT}$ $10\ln\left(\frac{5}{2}\right)=\ln\left(\frac{5}{2}\right)e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)T}$ $10=e^{\frac{1}{4}\ln\left(\frac{5}{2}\right)T}$ $\ln 10=\frac{1}{4}\ln\left(\frac{5}{2}\right)T$	1 for understanding initial rate and $\frac{dP}{dt}=40k$ when $t=T$ $\frac{dP}{dt}=10\times$initial RATE of increase, NOT initial price 10x10=100. This was a common error and gave an answer of 13.78
$T=\frac{\ln 10}{\frac{1}{4}\ln\left(\frac{5}{2}\right)}$ $=10.051766...$ ≈ 10.05 months OR	1 for solving

$\frac{dP}{dt}=k(P-6)$ When $t=0, P=10$ $\frac{dP}{dt}=k(10-6)$ $\frac{dP}{dt}=4k, initially$ Aim: to find T when $\frac{dP}{dt}=10\times 4k$ $\frac{dP}{dt}=10\times 4\times \frac{1}{4}\ln\frac{5}{2}$ $\frac{dP}{dt}=10\ln\frac{5}{2}$ $10\ln\frac{5}{2}=\frac{1}{4}\ln\frac{5}{2}(P-6)$ $40=P-6$ $P=46$ Sub into $P=4e^{kT}+6$ $46=4e^{kT}+6$ $40=4e^{kT}$ $e^{kT}=10$ $kT=\ln 10$ $T=\ln 10\div k$ $T=\ln 10\div\frac{1}{4}\ln\frac{5}{2}$ $T\cong 10\cdot 05176638...$ $T\cong 10.05 months$	
(b) (i) $\frac{dV}{dt}=-2, V=x^3,$ $\frac{dV}{dx}=3x^2$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ $-2=3x^2\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{-2}{3x^2}$	

Solutions	Marker's Comments
Question 13 (b) (ii) $\frac{dt}{dx}=\frac{3x^2}{-2}$ $-2dt=3x^2dx$ $\int -2dt=\int 3x^2dx$ $-2t=x^3+C$ $t=0, x=8$ $0=8^3+C$ $C=-512$ $-2t=x^3-512$ $x^3=512-2t$ $x=\sqrt[3]{512-2t}$	
(c) (i) $\frac{d}{dx}\left(\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}\right)=\frac{(4+x^2)\times 2-2x(2x)}{(4+x^2)^2}+\frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}$ $=\frac{8+2x^2-4x^2}{(4+x^2)^2}+\frac{2}{4+x^2}$ $=\frac{8-2x^2+2(4+x^2)}{(4+x^2)^2}$ $=\frac{8-2x^2+8+2x^2)}{(4+x^2)^2}$ $=\frac{16}{\left(4+x^2\right)^2}$	1 for <u>each</u> differentiation =2 1 for **showing** the required result
(c) (ii) $\int\frac{16}{\left(4+x^2\right)^2}dx=\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}+C$ $\int_0^{2\sqrt{3}}\frac{1}{\left(4+x^2\right)^2}dx=\frac{1}{16}\left[\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}\right]_0^{2\sqrt{3}}$ $=\frac{1}{16}\left(\frac{2(2\sqrt{3})}{4+(2\sqrt{3})^2}+\tan^{-1}\frac{2\sqrt{3}}{2}-\frac{2(0)}{4+(0)^2}-\tan^{-1}\frac{0}{2}\right)$ $=\frac{1}{16}\left(\frac{4\sqrt{3}}{16}+\frac{\pi}{3}-0-0\right)$ $=\frac{1}{16}\left(\frac{4\sqrt{3}}{16}+\frac{\pi}{3}\right)$ $=\frac{1}{16}\left(\frac{\sqrt{3}}{4}+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{64}+\frac{\pi}{48}=\frac{3\sqrt{3}+4\pi}{192}$	Some students missed the point of 'hence' ie. use the result from part (a) MUST be in radians. An answer of 3.78 comes from using degrees. Leave answer in exact form

Solutions	Marker's Comments
Question 14 (a) $f'(x) = \cot x + x$ $f(x) = \int \left(\frac{\cos x}{\sin x} + x \right) dx$ $= \int \left(\frac{\cos x}{\sin x} \right) dx + \int x dx$ $= \ln \lvert \sin x \rvert + \frac{x^2}{2} + C$ $f\left(\frac{\pi}{2}\right) = 0$ $\ln \lvert \sin \frac{\pi}{2} \rvert + \frac{\left(\frac{\pi}{2}\right)^2}{2} + C = 0$ $\ln 1 + \frac{\pi^2}{8} + C = 0$ $C = \frac{-\pi^2}{8}$ $f(x) = \ln \lvert \sin x \rvert + \frac{x^2}{2} - \frac{\pi^2}{8}$	Done well by most students. 37 got full marks, 9 got zero marks 1 for each integration Should have absolute value but no penalty was applied A few students incorrectly introduced a negative 1 for correct c and answer
(b) (i) $LHS = \cos 4x$ $= \cos x(2 \times 2x)$ $= 2\cos^2(2x) - 1$ $= 2(\cos 2x)^2 - 1$ $= 2\left(2\cos^2 x - 1\right)^2 - 1$ $= 2\left(4\cos^4 x - 4\cos^2 x + 1\right) - 1$ $= 8\cos^4 x - 8\cos^2 x + 2 - 1$ $= 8\left(\cos^4 x - \cos^2 x\right) + 1$ $= RHS$	Various methods accepted

Solutions	Marker's Comments
Question 14 (b) (ii) $\cos 4x = 8\left(\cos^4 x - \cos^2 x\right) + 1$ $\dfrac{\cos 4x - 1}{8} = \cos^4 x - \cos^2 x$ $\cos^2 x - \cos^4 x = \dfrac{1}{16}$ $\dfrac{1 - \cos 4x}{8} = \dfrac{1}{16}$ $1 - \cos 4x = \dfrac{1}{2}$ $\cos 4x = \dfrac{1}{2}, 0 \le x \le 2\pi$ $4x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$	NOT $1 - \dfrac{\cos 4x}{8} = \dfrac{1}{16}$ common error 1 mark 1 mark
(c) $a = 1, r = \dfrac{2x}{x+1}$ The series has a limiting sum when $\lvert r \rvert < 1$ or $-1 < r < 1$ $\left\lvert \dfrac{2x}{x+1} \right\rvert < 1$ $\lvert 2x \rvert < \lvert x+1 \rvert$ $4x^2 < (x+1)^2$ $3x^2 - 2x - 1 < 0$ $(3x+1)(x-1) < 0$ $-\dfrac{1}{3} < x < 1, x \ne 0$ **OR** $-1 < \dfrac{2x}{x+1} < 1$ $-1 < \dfrac{2x}{x+1}\ and\ \dfrac{2x}{x+1} < 1$ $-\left(x+1\right)^2 < \dfrac{2x}{x+1}\left(x+1\right)^2$ $-x^2 - 2x - 1 < 2x(x+1)$ $0 < 3x^2 + 4x + 1$ $(3x+1)(x+1) > 0$ $x < -1, x > \frac{-1}{3}$	Poorly done Many students tried to find the limiting sum rather than finding the values of x where a limiting sum will exist. Over 30 students did not even state the initial condition $\left\lvert \dfrac{2x}{x+1} \right\rvert < 1$ Various methods of solving the inequality were accepted

Solutions	Marker's Comments
Question 14 (d) (i) $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$ $= 2\underset{\sim}{a} + \overrightarrow{CD}$ $= 2\underset{\sim}{a} + \lambda\overrightarrow{CM}$ $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ $\underset{\sim}{b} = \underset{\sim}{a} + 2\overrightarrow{AM}$ $\overrightarrow{AM} = \dfrac{\underset{\sim}{b} - \underset{\sim}{a}}{2}$ $\overrightarrow{CM} = \overrightarrow{AM} - \overrightarrow{AC}$ $\overrightarrow{CM} = \overrightarrow{AM} - \underset{\sim}{a}$ $= \dfrac{\underset{\sim}{b} - \underset{\sim}{a}}{2} - \underset{\sim}{a}$ $= \dfrac{\underset{\sim}{b}}{2} - \dfrac{3}{2}\underset{\sim}{a}$ $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$ $\overrightarrow{OD} = 2\underset{\sim}{a} + \lambda\overrightarrow{CM}$ $= 2\underset{\sim}{a} + \lambda\left(\dfrac{\underset{\sim}{b}}{2} - \dfrac{3}{2}\underset{\sim}{a}\right)$ $= \left(2 - \tfrac{3}{2}\lambda\right)\underset{\sim}{a} + \tfrac{1}{2}\lambda\underset{\sim}{b}.$	Reasonably well done Students need to ensure that they give all working for 'show that' questions

(d) (ii) $\overrightarrow{OD} = \mu \underset{\sim}{b}$ $\overrightarrow{DB} = \underset{\sim}{b} - \mu \underset{\sim}{b}$ $\quad = (1-\mu)\underset{\sim}{b}$ $\overrightarrow{OD} + \overrightarrow{DB} = \underset{\sim}{b}$ $\left(2-\frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda \underset{\sim}{b} + (1-\mu)\underset{\sim}{b} = \underset{\sim}{b}$ $\left(2-\frac{3}{2}\lambda\right)\underset{\sim}{a} = \underset{\sim}{b} - \frac{1}{2}\lambda \underset{\sim}{b} - (1-\mu)\underset{\sim}{b}$ $\left(2-\frac{3}{2}\lambda\right)\underset{\sim}{a} = \left(\mu - \frac{1}{2}\lambda\right)\underset{\sim}{b}$ since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are not parallel or overlapping, for $\underset{\sim}{a} = \underset{\sim}{b}$, $\left(2-\frac{3}{2}\lambda\right) = 0$ and $\left(\mu - \frac{1}{2}\lambda\right) = 0$ $2 - \frac{3}{2}\lambda = 0, \lambda = \frac{4}{3}$ sub $\lambda = \frac{4}{3}$ into $\left(\mu - \frac{1}{2}\lambda\right) = 0$ $\mu - \frac{1}{2} \times \frac{4}{3} = 0$ $\mu = \frac{2}{3}$ $\overrightarrow{OD} = \frac{2}{3}\underset{\sim}{b}$ $\overrightarrow{OD} = \frac{2}{3}\overrightarrow{OB}$ $\frac{\overrightarrow{OD}}{\overrightarrow{OB}} = \frac{2}{3}$ $\frac{\overrightarrow{OD}}{\overrightarrow{OD}+\overrightarrow{DB}} = \frac{2}{3} = \frac{2}{2+1}$ $\therefore \overrightarrow{OD} : \overrightarrow{DB} = 2:1$	Poorly done Only 9 students got full marks for Q14d OR $\overrightarrow{OD} = \left(2-\frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda \underset{\sim}{b}$ from part (i) $\overrightarrow{OD} = \mu.\overrightarrow{OB} = \mu.\underset{\sim}{b}$ from part (ii) So $\left(2-\frac{3}{2}\lambda\right)\underset{\sim}{a} + \frac{1}{2}\lambda \underset{\sim}{b} = 0\underset{\sim}{a} + \mu \underset{\sim}{b}$ $\left(2-\frac{3}{2}\lambda\right) = 0$ and $\frac{1}{2}\lambda = \mu$ 1 for two equations linking λ and μ 1 for showing result